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Interelectron Scattering in Quasi-Intrinsic Germanium and Silicon*†

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The transient conductivity of germanium and silicon, induced by the generation of excess carriers by high-energy electrons, has been measured as a function of generation level. At the high generation levels (10^{16}–10^{17} carriers/cm^3), a striking reduction in the carrier mobility is observed both at 300 and 80°K. Specifically, for a generation level of 4×10^{16}/cm^3 in germanium at 80°K, the mobility is found to be decreased by a factor of 5 as a result of interelectron scattering. At 300°K, interelectron scattering is observed at higher generation levels because lattice scattering is relatively more important. A quantitative comparison with theory is made, with excellent agreement found.

INTRODUCTION

ELECTRON-ELECTRON scattering in extrinsic semiconductors becomes important either at normal temperatures, when the number of carriers is large (10^{16}–10^{17}/cm^3 in germanium and silicon), or at low temperatures, because the Rutherford scattering cross section for an electron with a thermal De Broglie wavelength is inversely proportional to the temperature squared.[1] The effect of interelectron scattering on the transport phenomena is usually more important when the electrons are distributed over two or more partly filled Bloch bands. In such a case, Coulomb scattering can occur between charge carriers each of which belongs to a *different* band. The transition probabilities are the same whether each of the two electrons stays in its original band or whether the two electrons exchange bands. Electron-hole scattering also reduces the mobility in intrinsic semiconductors like germanium or silicon. Physically, the difference between electron-electron and electron-hole scattering is simple: In the first case, the momentum conservation in a single scattering event implies the velocity conservation $\mathbf{v}_1+\mathbf{v}_2=\mathbf{v}_1'+\mathbf{v}_2'$, whereas, in the second case, the equality usually does not hold. In other words, electron-hole scattering processes affect the electrical conductivity primarily because of a current change in most scattering events and, secondarily, because energies are randomized. These processes, therefore, have a greater influence on the conductivity.

In intrinsic semiconductors, the carrier concentration can be increased by raising the temperature, but, simultaneously, the scattering of electrons and holes by the thermal lattice vibrations becomes more important. As a result, the relative influence of interelectron scattering on the mobility is small, being a maximum of 5% for intrinsic Ge. It is possible to produce a temperature-independent increase of several orders of magnitude in the carrier concentration of an homogenous semiconductor by exposing the crystal to high-energy primary electrons for short time intervals. Such a burst of ionizing radiation generates a large number of free electron-hole pair excitations as a result of the dynamical Coulomb interaction between primary electrons and crystal valence electrons. The conductivity process in a semiconductor exposed to such a short burst of ionizing radiation differs from a sample doped to the same conductivity level in that both signs of carriers are introduced, resulting in properties like those of an intrinsic specimen, and we have labeled these "quasi-intrinsic" specimens. As will be seen, one striking effect is in the reduction of the mobility by electron-electron and electron-hole scattering processes.[2] The experimental procedure and results, and a comparison of the results with theory will be given.

EXPERIMENTAL PROCEDURE

In this work, the material studied was n-type Ge, with initial carrier concentrations of 5×10^{13}/cm^3 and 1×10^{14}/cm^3 at 300°K, and n-type Si, with an initial carrier concentration of 1×10^{14}/cm^3 at 300°K. Bridge-type samples were studied, with the arms of the bridge employed as the voltage contacts and the ends of the sample used for attachment of the current leads. The sample was mounted in an aluminum can on an anodized surface to provide electrical insulation and to provide good thermal exchange with a coolant chamber on the other side of this surface.

The transient conductivity experiments were conducted using single pulses of 30-MeV electrons with pulse widths of 0.1 to 1 μsec in length. The 30-MeV electrons were supplied by the General Atomic electron linear accelerator, and the intensity of the beam was varied by changing the accelerator beam current which is monitored by a secondary emission foil at the output window.

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† A report on the preliminary results of this investigation was presented by J. Appel, R. Bray, and E. G. Wikner, in *Proceedings of International Conference on the Physics of Semiconductors, Exeter*, edited by A. C. Stickland (The Institute of Physics and The Physical Society, London, 1962).

[1] The Rutherford cross section includes the Debye screening length, which is also temperature-dependent. Consequently, the $(kT)^{-2}$ dependence is not exact.

[2] The influence of hole-hole scattering on the conductivity in p-type germanium has been measured by D. M. Brown and R. Bray, Phys. Rev. **127**, 1593 (1962), and analyzed by J. Appel and R. Bray, *ibid.* **127**, 1603 (1962).

In the experimental setup, two differential amplifier systems with push-pull outputs are used to monitor both the current and the voltage signals. They consist of Tektronix Type G inputs modified for remote gain and mode control, a main amplifier consisting of a replacement vertical amplifier for a Tektronix 545 oscilloscope, and a specially designed pair of high-current cathode-follower output stages which drives two 150-ft cables leading to the experiment room. This system has a rise time of 30 mμsec when utilized with a Tektronix 551 oscilloscope having another Type G differential preamplifier. The use of the push-pull feature of the main amplifier and cable driver minimizes rf noise in the measuring circuit.

The current to the sample was applied by a pulse generator which was turned on at the end of the accelerator ionization pulse. The amplifiers measured the voltage across the sample and the voltage across the 10-Ω series resistor after the carriers were injected.[3] The conductivity and carrier concentration were calculated from these data by using the standard formula

$$\sigma=\frac{I}{V}\frac{l}{A}=ne(\mu_n+\mu_p),$$

where the symbols have their usual meaning. To determine the carrier concentration at the low generation levels, the carrier mobility was taken to be equal to the lattice mobility. In germanium, the sum of the electron and hole mobilities used was 5600 cm^2/V-sec at 300°K and 80 000 cm^2/V-sec at 80°K. In silicon, the sum of the room temperature mobilities was taken as 1900 cm^2/V-sec. It is assumed that the lattice mobility does not depend on the number of carriers (up to 10^{17}/cm^3), and the reduction of the carrier mobility at the high generation levels is postulated to be due to interelectron scattering.

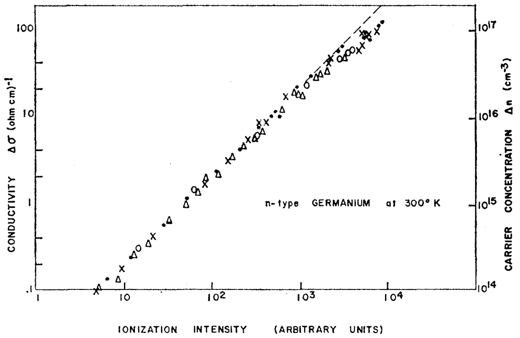

FIG. 1. Transient conductivity, $\Delta\sigma$, in germanium at 300°K as a function of ionization intensity. Data are shown for four different samples. The influence of interelectron scattering is seen to be clearly evident at a carrier concentration of 5×10^{16}/cm^3. The dashed line is an extrapolation from the low intensity results.

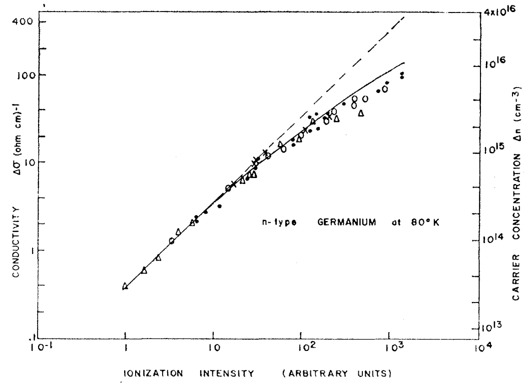

FIG. 2. Transient conductivity in germanium at 80°K versus ionization intensity. Data are shown for four samples, and the influence of intercarrier scattering is seen to occur at lower generation levels than at 300°K. The solid line is the theoretical prediction of interelectron scattering on the conductivity, and the dashed line is the extrapolation from the low intensity results.

The time delay between the accelerator ionization pulse and the applied voltage pulse was less than 0.5 μsec, which was short compared with the recombination lifetimes observed. Corrections were made for the recombination of carriers in this time interval by extrapolating the observed voltage decay to the middle of the ionization pulse. The probable error in the experiment was dependent on the generation level because, experimentally, the excess carrier lifetimes are found to be dependent on generation level.[4] At low levels, the maximum error is about 5%. At medium levels, where the conductivity versus generation level begins deviating from linearity, the error was between 10 and 20%. At the highest levels, the error was less than 5%. The highest generation level reached was 3×10^{17} carriers/cm^3.

EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

The change in electrical conductivity which occurs in single crystals of germanium at 80 and 300°K and of silicon at 300°K has been measured as a function of generation level, and the influence of interband and intraband scattering processes has been demonstrated. Here, by intraband scattering, we mean electron-electron, heavy hole-heavy hole, and light hole-light hole; and, by interband scattering, we mean electron-heavy hole, electron-light hole, and heavy hole-light hole. Figures 1 and 2 show the results for germanium at 300 and 80°K, respectively. The 300°K data show a linear conductivity change up to high generation levels (about 5×10^{16}/cm^3); the measured curve then bends away from the linear line, and it appears this deviation

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[3] Another experimental method also was used at the low generation levels. In this case, dc current was applied to the sample and the amplifiers measured the change in voltage and current during an ionization pulse. There is a considerable conductivity range in which both methods are applicable and consistent results were obtained.

[4] A complete report on this observation will be published in the near future: cf., also W. Shockley and W. T. Read, Phys. Rev. **87**, 835 (1952).

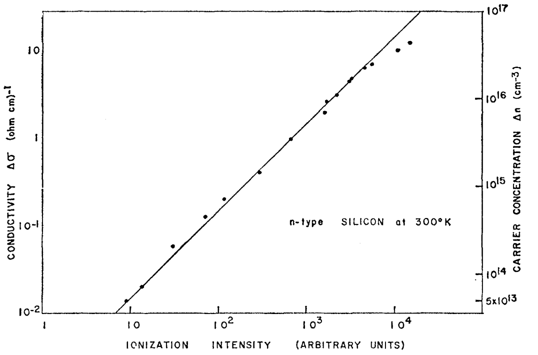

FIG. 3. Transient conductivity in silicon at 300°K versus ionization intensity. Note that the influence of interelectron scattering occurs at lower carrier concentrations than for germanium at 300°K.

is caused mainly by the scattering of charge carriers by one another.[5] At 80°K, the effect is seen to occur at much lower generation levels (approximately $10^{15}/\text{cm}^3$). The reason for this is that, in intrinsic semiconductors at high temperatures where the electron-hole pair concentration is the same order of magnitude as in this experiment, the dynamical interaction between electrons and holes is weak compared with their lattice interaction. This is true because the contribution of lattice scattering and of carrier-carrier scattering to the electrical resistivity, respectively, increases and decreases with temperature. The striking influence on the mobility at 80°K is then not surprising. The question as to whether the excess carriers are being generated linearly can be answered by comparing the 300 and 80°K results. At 300°K, the slope of the conductivity curve is still linear at the highest generation levels reached at 80°K ($4\times10^{16}/\text{cm}^3$). It is inconceivable that carrier generation should be temperature dependent in this region. Also, the thermal heating of the sample by the primary electrons has been measured and found to be, at most, 1°K at the highest levels. This causes a maximum error in the measurements of 2%.

The experimental results were compared with the theory of Appel[6] in which the total conductivity of germanium is calculated as a function of electron-hole pair concentration at 80 and 300°K, incorporating acoustical and optical phonon scattering together with carrier scattering; impurity scattering is neglected. Of all different types of intercarrier scattering, electron-hole scattering is most important since the conservation of momentum in an electron-hole collision does not, in general, result in the conservation of the current vector. The comparison between the experimental values and the calculated curve[6] (see Fig. 2) at 80°K shows good agreement with the exception of the highest carrier concentrations where the experimental points lie below the theoretical curve. This deviation is to be expected. The assumption of plane wave states for the conduction band electrons and for the valence band holes is, for our purposes, a fair assumption *only* in the immediate neighborhood of the energy extrema. In particular, the warping of the valence bands has been neglected. Furthermore, the theoretical considerations are based on a screened Coulomb interaction of the form

$$V(r)=\pm\frac{e^2}{\kappa r}e^{-r/\lambda_D},$$

where κ is the static dielectric constant, and λ_D is the Debye screening length. For *large* carrier concentrations, this two-particle interaction underestimates the strength of the actual interaction, since the dielectric screening caused by the polarization of the lattice is overestimated by the static dielectric constant. At 300°K, the experimental points also lie below the theoretical curve at large carrier concentrations ($>8\times10^{16}/\text{cm}^3$). Quantitative comparison is difficult in the region where the theory applies because only a small influence on the mobility is expected (i.e., 4% at 10^{16} carriers/cm^3 and 13% at 10^{17} carriers/cm^3), and also because a relatively large experimental error (10–20%) is caused by the fast recombination time at these generation levels.

In silicon, it is expected that the effect of interelectron scattering processes on the mobility will occur for smaller carrier concentrations than for germanium. One obvious reason for this is the different strengths of the screened Coulomb interaction between charge carriers in germanium and silicon. The Born-approximation scattering cross section for Coulomb scattering of charge carriers is inversely proportional to κ^2, and, therefore, the Coulomb interaction in silicon is larger by a factor of 2 or 3 than it is in germanium. Also, the ratio of effective masses, which occurs in the theory, is different in the two cases. Figure 3 shows the results of the transient conductivity change for silicon at 300°K. The influence of interelectron scattering on the mobility is, indeed, seen at lower electron-hole pair concentrations for silicon.

The interconsistency of our results, the comparison of germanium and silicon at 300°K, the comparison for germanium at 80 and 300°K, and the excellent agreement between theory and experiment for germanium at 80°K, lead us to believe that the effect of interelectron scattering on the transient mobility has been clearly demonstrated.

ACKNOWLEDGMENTS

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[5] L. W. Davies, Nature **194**, 762 (1962), observed the mobility decrease in the central, lightly doped (*L*) region of a p^+-L-n^+ structure by injecting carriers with a current pulse in the forward direction. A strong influence on the mobility is seen for carrier densities an order of magnitude below that seen in this experiment. The reason for this difference is not understood.

[6] J. Appel, Phys. Rev. **122**, 1760 (1962); **125**, 1815 (1962).